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Tutorial

By Prof. dr Srdjan Stankovic, University of Montenegro

Compressive sensing: Theory, Algorithms and Applications

Abstract

In the era of technology expansion, the digital devices are made to achieve high resolution signal acquisition, producing a large amount of digital data. This is a common issue in sensing systems dealing with radar signals, multimedia signals, medical and biomedical data, etc. The acquisition process is done according to the Shanon-Nyquist theorem with the sampling rate which is usually at least twice the highest signal frequency. Consequently, in order to respond to the storage, transmission and computational challenges, such acquired data are compressed up to the acceptable quality. Obviously by using compression, we are able to significantly reduce the amount of digital data keeping a high level of decoded signal quality. This reduced form of data has a positive impact in the sense of hardware and infrastructure requirements, which opened an uncountable number of applications. However, the data acquisition process is still demanding in terms of resources (e.g. sensors technology) and acquisition time. In recent years, the compressive sensing approaches have been intensively developed with the idea to overcome the limits of traditional sampling theory and to apply a concept of compression during the sensing procedure. In that sense, significant efforts have been done toward the development of methods that would allow to sample data in the compressed form using much lower number of samples. In this tutorial the basic concept of compressive sensing will be considered. The compressive sensing theory states that the signal can be reconstructed using just a small set of randomly acquired samples if it has a sparse (concise) representation in certain transform domain. In other words, since most of the real-life signals have compressible representation with just a small number of non-zero coefficients, signals can be reconstructed using much fewer samples than required by the traditional sampling theorem. Another important condition, apart from the signal sparsity, is incoherence between the measurement process and the sparsity basis. More incoherence means fewer measurements. To that end, these important properties: signal sparsity, restricted isometric property and incoherence will be discussed. The full signal reconstruction is formulated as a problem of solving undetermined system of linear equations using sparseness constraints. There are several standard algorithms that could be employed for this purpose. For

instance, the constrained *l*1-minimization has been used as one among first approaches for finding the sparse solutions and it is known as basis pursuit. Alternative approaches are called greedy algorithms and among them the most popular is the iterative Orthogonal Matching Pursuit (with a variety of modifications). On the other side, the most recent method for CS reconstruction is based on the noise model describing the missing samples effect influencing sparsity domain. The algorithm distinguishes and reconstructs signal among various non-signal components in a single or few-iteration process. Another method reviewed here, is derived from the postulates of robust estimation theory. It introduces a new minimization metrics called generalized deviations, which is defined as the *p*-th norm of the total error, where *p* arises from the noise distribution. Therefore, this approach effectively reduces the influence of noise to the reconstruction performance. In the conditions of approximate sparsity, the gradient based algorithm is defined to comply with various types of signals and transform domains. This method is efficient for both 1D and 2D data, including the highly demanding natural images. Finally, the generalization of the entire concept can be extended for some other domains, not so typical in CS literature but quite important for signals in real applications, such as polynomial Fourier and Hermite transform domain.

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